

California High School Exit Examination

Mathematics Teacher Guide



California Department of Education
2004

California High School Exit Examination Mathematics Teacher Guide

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Foreword to the 2004 Edition of the CAHSEE Mathematics Teacher Guide

This year's edition of the Teacher Guide has been updated but not changed substantially from the 2002 edition. The updates include:

- New information has been provided about the consequences of the test, based on the ruling of the State Board of Education in July 2003 (see page 2).
- Ten released test questions are new to this edition of the Teacher Guide and are provided as samples for the following mathematics content standards:

Number Sense 2.2

6th Grade Statistics, Data Analysis, and Probability 2.5

Algebra and Functions 4.2

Measurement and Geometry 1.1

Measurement and Geometry 2.3

Measurement and Geometry 2.4

Measurement and Geometry 3.3

Algebra I 2.0

Mathematical Reasoning 2.1

Mathematical Reasoning 2.4

(see pages 21, 27, 50, 53, 60, 61, 64, 68, 82, and 85).

The released test questions for Algebra and Functions 4.2 and Algebra I 2.0 are from the 2002 set of released test questions. The released test question for each of the other standards noted above are from the 2004 set of test questions released this Spring.

- Additional information about resources available for teachers has been provided (see pages 9 and 10).

In addition, the question and answer section and the test blueprints have been removed from the Teacher Guide, as this information is available on the CDE Web site:

<http://www.cde.ca.gov/ta/tg/hs/>

Introduction

The California High School Exit Examination (CAHSEE) Teacher Guides for English-language arts (ELA) and mathematics are designed to provide comprehensive and accessible information to assist teachers in preparing their students for the CAHSEE. This guide contains two sections, and teachers are encouraged to reproduce individual sections or all of the guide for classroom use. Districts and school personnel also are encouraged to use this material in their staff development activities.

- **Section 1**, “About the CAHSEE,” provides an overview of the purpose and content of the CAHSEE. This section also includes a set of checklists teachers may use when writing test questions for classroom use, information about security for the CAHSEE, and a glossary of terms used in this guide.
- **Section 2**, “Assessing the CAHSEE Mathematics Standards,” is designed to give detailed information about how the California academic content standards for mathematics are tested on the CAHSEE. The mathematics standards assessed on the CAHSEE are from the following strands:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Algebra I
- Mathematical Reasoning

Section 2 provides a summary of the essential knowledge and skills covered in each strand, followed by information about how the standard may be tested. For each standard, a released test question from a previous administration of the CAHSEE is provided, with an explanation of the correct answer and an analysis of the incorrect answers.

Section 1

About the CAHSEE

Background

After determining that local proficiency standards established pursuant to Education Code Section 51215 (repealed January 1, 2000) were generally set below a high-school level and were not consistent with the state's academic content standards, the Legislature indicated its intent to set higher standards for high school graduation. In proposing the California High School Exit Examination (CAHSEE), the Legislature's primary goal was to "... significantly improve pupil achievement in high school and to ensure that pupils who graduate from high school can demonstrate grade level competency in reading, writing, and mathematics . . ." (Senate Bill 2, Section 1[b]). Education Code Section 60850 (Chapter 1, statutes of 1999-2000, S.B. 2X, O'Connell) authorized the CAHSEE to be developed in accordance with State Board of Education (SBE)-adopted academic content standards in English-language arts and mathematics. The CAHSEE was developed based on recommendations of the High School Exit Examination Standards Panel, whose members were appointed by the State Superintendent of Public Instruction and approved by the SBE.

The CAHSEE was offered for the first time in spring 2001 (March and May) to volunteer 9th graders (class of 2004). In October 2001, Assembly Bill 1609 (Calderon) removed the option for 9th graders to take the CAHSEE beginning with the 2002 administration. The CAHSEE was next administered in spring 2002 to all 10th graders who had not passed it during the spring 2001 administration. It has since been administered several times to the remaining students in the class of 2004 who had not yet passed one or both parts (i.e., ELA and mathematics). The class of 2005 took the CAHSEE for the first time in the spring of 2003. In July 2003, the SBE took action to move the passage of the CAHSEE as a condition of graduation to the Class of 2006. Beginning in the 2005-2006 school year, students must pass the CAHSEE as a condition of graduation, as well as meet the district's requirements for graduation. The Class of 2006 took the CAHSEE for the first time as 10th graders in February or March 2004.

For more information about the CAHSEE, visit the CDE Web site at <http://www.cde.ca.gov/ta/tg/hs/>

Purpose and Content

The primary purpose of the CAHSEE is to significantly improve pupil achievement in public high schools and to ensure that pupils who graduate from public high schools can demonstrate grade-level competency in reading, writing, and mathematics. The CAHSEE helps identify students who are not developing skills that are essential for life after high school and encourages districts to give these students the attention and resources necessary to help them achieve these skills during their high school years.

The CAHSEE has two parts: English-language arts (ELA) and mathematics. The ELA part addresses state academic content standards through grade 10. In reading, this includes vocabulary, decoding, comprehension, and analysis of information and literary texts. In writing, this covers writing strategies, applications, and the conventions of English (e.g., grammar, spelling, and punctuation). The mathematics part of the CAHSEE addresses state academic content standards in grades 6 and 7 and Algebra I. The exam includes statistics, data analysis, and probability; number sense; measurement and geometry; mathematical reasoning; and algebra. Students are also asked to demonstrate a strong foundation in computation and arithmetic, including working with decimals, fractions, and percents.

The test questions on the CAHSEE assess a range of difficulty levels, consistent with good testing practices. These questions assess full mastery of the designated academic content standards as well as foundational knowledge and skills underlying these standards, as recommended by the High School Exit Examination Standards Panel.

For example, the ELA part of the CAHSEE includes test questions that require students to determine the meaning of words in context. This vocabulary strategy is not specifically named in the grades 9 and 10 academic content standards, but it is included on the exam because it is a foundational, underlying skill required for achievement of the grades 9 and 10 vocabulary standards. Similarly, most CAHSEE ELA test forms contain at least one poem. Although poetry is not specifically named in the grades 9 and 10 literary analysis standards, analysis of poetry is a focus of standards in the earlier grades. In mathematics, standards from grades 6 and 7 and Algebra I are included on the CAHSEE because these academic content standards represent both foundational and competency standards that students should meet to graduate from high school.

All questions on the examination have been evaluated for their appropriateness for measuring the designated ELA and mathematics academic content standards. They have been reviewed and approved by committees of California educators, including teachers, administrators, and academicians. In addition to being reviewed for content, all items have been reviewed and approved by California educators for their adherence to the principles of fairness and have been evaluated to determine if bias exists with respect to characteristics, such as gender, ethnicity, and language.

Both parts of the CAHSEE (ELA and mathematics) have multiple-choice questions, which consist of a question or statement followed by a set of four possible answer choices. Only one answer choice is correct, and there is no scoring penalty for guessing an answer choice incorrectly. Mathematics items may include pictorial material, such as drawings, tables, diagrams, or graphs. In addition to multiple-choice questions, there is a writing task on the ELA part of the CAHSEE. The task requires either a response to literary or informational text or a response to a stand-alone writing prompt.

Each operational form of the mathematics portion of the CAHSEE includes 12 field-test questions consisting of newly developed items. The ELA portion of the CAHSEE includes 7 field-test questions (multiple-choice only) on each operational form. The field-test questions are not identified in the test booklets. These questions are selected for the collection of statistical data only and are not included in any individual or group student score reports.

The test blueprints for the CAHSEE, which indicate the academic content standards tested and the number of items per standard, are available on the CAHSEE Web site (noted previously in the Background section).

The CAHSEE is not a timed test, which means it has no fixed time limit in which students must complete the examination. However, students are expected to complete their work during the regular school day unless their Individualized Educational Program (IEP) or Section 504 Plan specifies the need for extra time beyond the school day.

English-Language Arts Academic Content Standards

The standards for the English-language arts part of the CAHSEE are taken from the California academic content standards for grades 9 and 10. Standards from the following strands are included: Word Analysis, Reading Comprehension, Literary Response and Analysis, Writing Strategies, Writing Conventions, and Writing Applications.

The English-language arts part of the CAHSEE contains 79 multiple-choice test questions (72 operational questions and 7 field-test questions) and 1 writing task. Table 1 shows how the operational items are distributed across the six strands.

Table 1
Distribution of CAHSEE ELA Questions by Strand

Strand	Number of Multiple-Choice Items	Number of Writing Tasks
Word Analysis	7	-
Reading Comprehension	18	-
Literary Response and Analysis	20	-
Writing Strategies	12	-
Writing Conventions	15	-
Writing Applications	-	1
Total	72	1

Mathematics Academic Content Standards

As mentioned above, the standards for the mathematics part of the CAHSEE are taken from the California academic content standards in grades 6 and 7, and in Algebra I. They include standards from the following mathematical strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Algebra I; Mathematical Reasoning.

The mathematics part of the CAHSEE contains 92 multiple-choice test questions (80 operational questions and 12 field-test questions). Table 2 shows how the operational items are distributed across the six strands.

Table 2
Distribution of CAHSEE Mathematics Questions by Strand

Strand	Number of Multiple-Choice Items
Number Sense	14
Statistics, Data Analysis, and Probability	12
Algebra and Functions	17
Measurement and Geometry	17
Algebra I	12
Mathematical Reasoning	8
Total	80

Development of Test Questions for the CAHSEE

The test questions that appear on the CAHSEE have been through an extensive development process to ensure that they are valid and fair measures of what students know and are able to do.

Content Validity

To ensure that the CAHSEE is a valid measure of the specified academic content standards, the test questions are carefully designed to assess the content indicated in the test blueprints. Insofar as possible, each question requires students to demonstrate knowledge and/or skills in only one standard. Because many academic content standards cover a wide range of knowledge and skills, individual test questions may assess one component of the standard. Other questions may address underlying, foundational knowledge or skills that are required for higher achievement in the standard.

Technical Quality

Well-written test questions give students an opportunity to demonstrate what they know and are able to do; students do not have to guess what the question is asking. When questions are clearly written and easily understood, students are able to provide evidence of their learning. Test questions have only one clearly correct answer. The language is simple, direct, and free of ambiguity. Questions should not test reading ability or vocabulary if that is not the purpose. CAHSEE test questions are reviewed for content validity and technical quality by committees of California educators.

Test Bias

Bias in testing can take several forms, including the use of unfamiliar or insensitive language and terms, the presentation of stereotypes, and the inclusion of concepts that are offensive or negative toward any group. During the development process, CAHSEE test questions are continually reviewed for potential bias to ensure that the CAHSEE meets the highest professional testing standards.

For general matters of style and grammar, CAHSEE test developers consult *The Chicago Manual of Style*, *The Gregg Reference Manual*, *Merriam Webster's Collegiate Dictionary*, and *The American Heritage College Dictionary*.

The following checklists are used by CAHSEE item writers and review committees as a basis for evaluating the content validity, technical quality, and fairness of test questions. Teachers may also use these checklists to improve their own classroom assessments. For multiple-choice questions for either English-language arts or mathematics, teachers may wish to write standards-based test questions to help students prepare for the CAHSEE. The checklist in Table 3 is provided for teachers to evaluate their own questions against the general requirements for CAHSEE multiple-choice questions. Table 4 provides some useful guidelines teachers may use for developing English-language arts writing tasks.

Table 3
Development Checklist for Multiple-Choice Questions

✓	Requirements
The test question as a whole—	
	Has one and only one clearly correct answer
	Clearly presents one central idea
	Measures the intended objective
	Has a clear purpose
	Is within the appropriate range of difficulty
	Contains simple, direct, and unambiguous language
	Uses age-appropriate vocabulary and sentence structure
	Does <u>not</u> use vocabulary and idiomatic phrases that could be unfamiliar
	Does <u>not</u> rely on students' possessing outside knowledge
	Tests worthwhile (not trivial or obscure) concepts or information
	Reflects current teaching practices
	Is <u>not</u> tricky or cute
	Does <u>not</u> appear to ask for the student's opinion
	Is grammatically correct
	Uses the active voice and avoids informal diction and usage
	Follows the appropriate style guidelines
	Is free of bias, sensitive language or topics, and stereotypes

Development Checklist for Multiple-Choice Questions, continued

✓	Requirements
The stem of the test question—	
	Gives the test taker a full sense of what the item is asking
	Is either a question or an incomplete statement
	Is both clear and concise
	If negative, contains no negatives in the distractors
The stimulus or passage for the test question(s)—	
	Is likely to be interesting to students
	Is correctly and clearly labeled
	Provides all the information needed to answer the questions
	Can be reproduced clearly in a test book
The response options—	
	Are written so that no one option is significantly different from the others in length, specificity, or complexity
	Relate to the stem in the same way
	Do <u>not</u> include an option that denies the truth of any other option
	Do <u>not</u> deny the truth of the stem
	Do <u>not</u> give clues to students, such as the use of absolutes like <i>always</i> and <i>never</i>
	Do <u>not</u> repeat words that could be placed in the stem
	Include plausible and reasonable misconceptions and errors
	Do <u>not</u> include distractors that are phrased differently but have the same meaning as other distractors
The set of test questions—	
	Includes a sufficient number of questions to justify the time required to read the stimulus or passage
	Contains questions that are entirely independent of each other
	Reflects an appropriate range of difficulty

Table 4
Development Checklist for Writing Tasks

✓	Requirements
The task—	
	Directly assesses the knowledge and/or skills specified by the academic content standard
	Clearly tells students what they are being asked to do
	Is appropriate in scope, i.e., neither too broad nor too narrow
	Uses precise action verbs and descriptive words
	Invites and supports a range of responses
	Is specific about the expected level of detail required in the response
	Does <u>not</u> invite personal responses about students' values or beliefs
	Does <u>not</u> advocate a particular value that may not be common to all students
	Uses age-appropriate vocabulary and sentence structure
	Does <u>not</u> use unfamiliar vocabulary or unfamiliar idiomatic phrases
	Is free from problems of bias or sensitivity
	Is likely to be a topic of interest to students

Test Security

One of the most significant guarantors of fairness to all students who take the CAHSEE is that passages, writing prompts, graphical materials, and test questions remain secure at all times. Individuals who circumvent or attempt to circumvent procedures to maintain test security diminish the legitimate and honest efforts of all other students and teachers to participate in the state's assessment system. The California Department of Education (CDE) has the authority, according to the Education Code Section 60851 (b) and (c) and the copyright statutes of the United States, to act against any individual or group of individuals who knowingly attempt to copy, duplicate, or transmit in any way, the contents of secure material from test booklets, answer documents, in whole or in part, to any other individual or group of individuals. The California Department of Education may employ procedures to maintain the test security of the CAHSEE, including but not limited to monitoring of test administration, document handling, and post-test analytic techniques such as mark discrimination analysis.

Student Study Guides

In November 2003, CDE released study guides for each portion of the CAHSEE, English-language arts and mathematics. The study guides feature answers to frequently asked questions, test-taking tips, and practice tests. Each guide dedicates one chapter to each strand, and includes released test questions with the solutions explained. While the study guides were written for students to use independently, teachers may incorporate them into their classroom instruction to prepare their students for the CAHSEE. CDE will provide each tenth grade student in the class of 2006 with a hard copy of each study guide. Additionally, the guides are available on the CDE Web site.

Resource Documents

The information in this Teacher Guide is based on the California academic content standards and the California frameworks in English-language arts and mathematics. These documents may be ordered from the California Department of Education, or they may be downloaded from the CDE Web site, as shown below:

The English-Language Arts Content Standards for California Public Schools, Kindergarten Through Grade Twelve is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

The Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve (1997) is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

The Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (1999), is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

The Reading/Language Arts Framework for California Public Schools, Kindergarten Through Grade Twelve is available from the California Department of Education, CDE Press, Sales Unit, 1430 N Street, Suite 3207, Sacramento, CA 95814; 1-800-995-4099, ext. 1. It is also available at <http://www.cde.ca.gov/> on the Internet.

Other Resources

The student study guides for the CAHSEE are available at
<http://www.cde.ca.gov/ta/tg/hs/resources.asp>

Released Test Questions from the CAHSEE are available at
<http://www.cde.ca.gov/ta/tg/hs/resources.asp>

Answers to Frequently Asked Questions (FAQs) are available at
<http://www.cde.ca.gov/ta/tg/hs/faq.asp>

Blueprints for the CAHSEE are available at
<http://www.cde.ca.gov/ta/tg/hs/admin.asp>

Glossary of Terms Used in this Guide

Answer Choices — The correct answer and the distractors in a multiple-choice test question.

Blueprint — The plan for assessment that specifies the number of questions on each test form according to strand and academic content standard.

Clueing — An instance in which one test question provides information that could be used to select the correct answer to another question, or an instance in which the stem in a multiple-choice question clues the correct answer.

Constructs — The underlying cognitive domains for each strand in the California academic content standards (e.g., conceptual understanding or problem-solving in mathematics; reading, understanding, and analyzing grade-level texts in English-language arts).

Distractors — Incorrect answers to a multiple-choice stem.

Field-Test Questions — Test questions that are administered to students to gain information about the quality of the question. Student performance on these questions does not affect student scores.

Foundational Knowledge/Foundational Skill — Knowledge or skill that a student would be taught and be expected to know prior to taking courses covering the academic content standards tested in the CAHSEE.

Item — A test question written in one of several possible item formats.

Item Format — The basic design of a test question (e.g., multiple-choice, constructed response).

Key — The correct answer to a multiple-choice question.

Multiple-Choice Question — A stem plus a number of response options or answer choices (four for CAHSEE).

Response Options — The choices in a multiple-choice question, consisting of one key (correct answer) and a number of distractors (three for CAHSEE).

Scoring Guide — The rubric or protocol to follow when assigning a point value to responses to a writing task.

Specifications — The document that includes a description of how each standard is assessed on the CAHSEE.

Standard — Statement of what students should know and be able to do.

Stem — The initial part of a multiple-choice test question in which the task or premise is given. The stem may be a question, an incomplete statement, or a set of directions.

Stimulus — A picture, graph, map, chart, quotation, or other text that students are asked to interpret when answering a test item.

Strand — A category of standards that relate to each other for purposes of reporting performance on the CAHSEE.

Writing Task — A test question in which students are asked to supply their own response to a question rather than choose among options for a correct answer.

Section 2

Assessing the CAHSEE Mathematics Standards

The Mathematics part of the California High School Exit Examination (CAHSEE) assesses designated California academic content standards from grades 6 and 7 and Algebra I. A multiple-choice format is used to assess six strands: Number Sense; Statistics, Data Analysis, and Probability; Algebra and Functions; Measurement and Geometry; Algebra I; and Mathematical Reasoning. Each of these strands is described in detail in the following section of the Teacher Guide. For reporting purposes, the Statistics, Data Analysis, and Probability strands for grades 6 and 7 are combined. The Mathematical Reasoning test questions, which are always based on concepts in Number Sense, Statistics, Data Analysis, and Probability, Algebra and Functions, and Measurement and Geometry, are reported under those strands. Thus, there is no reporting category specifically for Mathematical Reasoning.

The CAHSEE focuses on mathematics constructs that are taught and assessed throughout elementary, middle, and high school. Three underlying constructs have been identified for the mathematics part of the CAHSEE: computational and procedural skills, conceptual understanding, and problem solving. Test questions on the exam cover one or more of these constructs, and CAHSEE test question writers and reviewers verify that each question measures the appropriate construct as well as the identified academic content standard. The constructs for each standard are given in the following pages.

Although test questions for the mathematics part of the CAHSEE do not specifically test students on mathematics vocabulary, they may require students to understand mathematical terms. It is especially important that students know the terms that appear in the language of the academic content standards associated with a question.

The following pages of the Teacher Guide discuss the mathematics strands and academic content standards included in the CAHSEE. The mathematics strands are:

- Number Sense
- Statistics, Data Analysis, and Probability
- Algebra and Functions
- Measurement and Geometry
- Algebra I
- Mathematical Reasoning

After each strand is described, each standard in that strand is discussed in detail, and a sample released test question is provided to illustrate each standard. Also included are possible reasons that students might select the distractors, as well as an explanation of the correct answer. Teachers will find this section of the guide useful in understanding how the California academic content standards are assessed on the CAHSEE. A thorough understanding of the standards and the test questions associated with them will help teachers focus their instruction on the academic content standards and better prepare students for the exam.